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13. ABSTRACT (Maximum 200 words)

The objective of this research is the development of systematic methods for generating reduced-order models that accurately describe the vibrations of large-scale nonlinear structural systems. These methods are based on nonlinear modes of vibration defined and constructed in terms of invariant manifolds. The motivation for the research stems from the fact that the dynamics of nonlinear structures are typically decomposed in terms of the linearized system's modes, often yielding poor modal convergence and too large reduced-order models. Research focused on (1) the development of a more efficient computer code for the generation of reduced-order models for a realistic rotorcraft blade model, (2) the investigation of the convergence of reduced-order models based on a finite-element rotorcraft blade model, (3) the extension of the model reduction technique to piecewise linear systems and hybrid linear-nonlinear structural systems, (4) the development of a new computer code for the construction of multi-mode based reduced-order models, (5) the development of a new computer code for the construction of the reduced-order model for nonlinear systems under periodic external excitation, (6) the development of a methodology that uses component mode synthesis in conjunction with nonlinear normal modes.

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Model Reduction Techniques for Large-Amplitude Vibrations of Complex Nonlinear Structures

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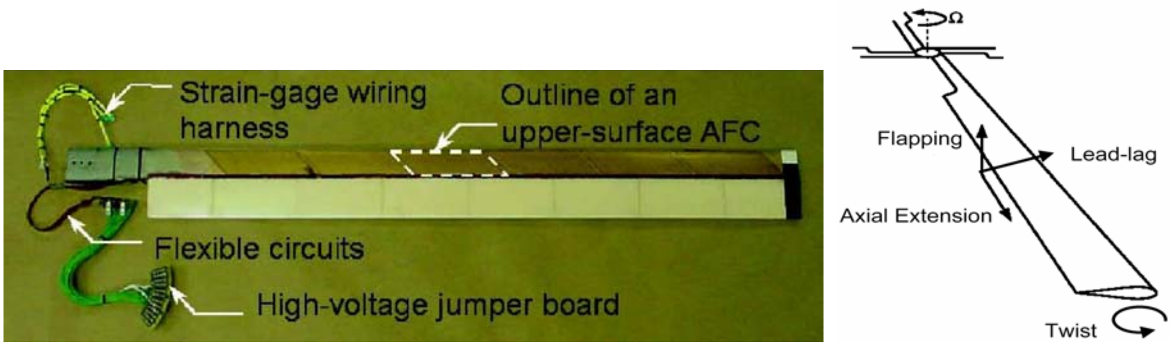
1 STATEMENT OF THE PROBLEM STUDIED

The goal of this project is to develop systematic methods for generating reduced-order models that accurately describe the vibrations of large-scale nonlinear structural systems, and to apply these methods to systems of interest to ARO, such as rotorcraft blades. The general approach is a type of *nonlinear modal analysis*, which makes use of nonlinear normal modes that are defined in terms of invariant manifolds in the phase space of the system model. In previous work, a new solution technique, namely a *Galerkin projection method*, was developed to construct nonlinear normal modes that are accurate out to large amplitudes of vibration. Using the minimally sized models obtained by this new method, we can effectively predict the vibration of structural components, joints, and assemblies. We have been concentrating on improving the efficiency of this new solution technique, and have successfully applied this reduction method to a realistic rotorcraft blade model. Finally, we have developed a nonlinear component mode synthesis technique that allows one to generate reduced order models for complex systems composed of multiple subsystems by making use of nonlinear modal representations of the substructures. These achievements are summarized below.

2 SUMMARY OF THE MOST IMPORTANT RESULTS

2.1 Reduced-order modeling of a realistic rotorcraft blade model

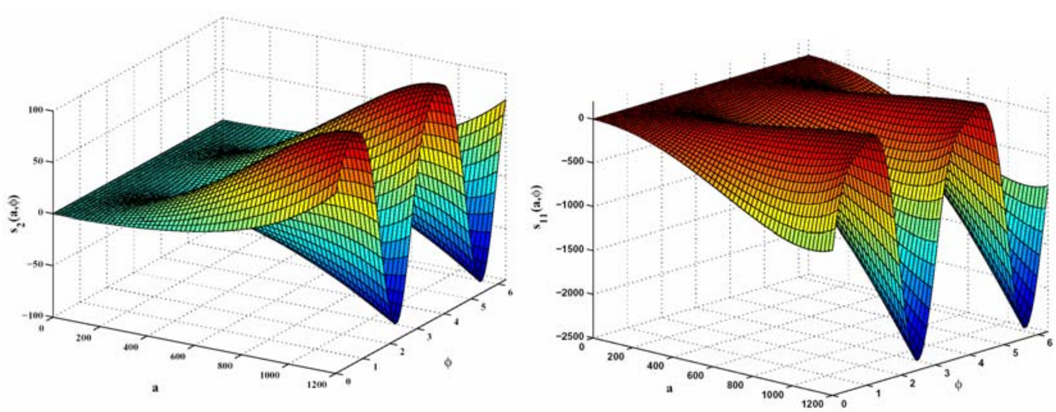
In previous research effort, the methodology of nonlinear modal analysis had been applied to a simplified rotating blade model, where only the transverse flapping motion and the axial elongation movement were considered. A breakthrough has been achieved by applying the nonlinear-normal-mode-based model reduction methodology to a *rotating rotorcraft blade*. The system considered is an active twist rotor (ATR) blade. The blade model is based on a large-deflection beam theory, and includes various beam motions such as bending, torsion, lead-lag, and axial elongation. The resulting discretized finite element model accounts for geometrically large deformation. A nonlinear-normal-mode-based reduced-order model has been constructed to simulate the hovering motion of the ATR blade. The nonlinear mode approach allows one to reduce the nonlinear dynamics of a large-scale FE-based model to a single degree of freedom that accurately captures the dominant dynamics of the blade. This is accomplished by systematically slaving all blade degrees of freedom to the response of dominant flapping motion. The resulting single-degree of freedom reduced-order model enables the accurate, fast simulation of the ATR blade dynamics, and it is valuable in understanding the dynamic behavior of rotorcraft blades undergoing large deformations.



(a) (b)

Figure 1. The ATR blade model. (a) The prototype of the active twist rotor (ATR) blade [1]. AFC denotes the active fiber composites. (b) Motions of the ATR blade.

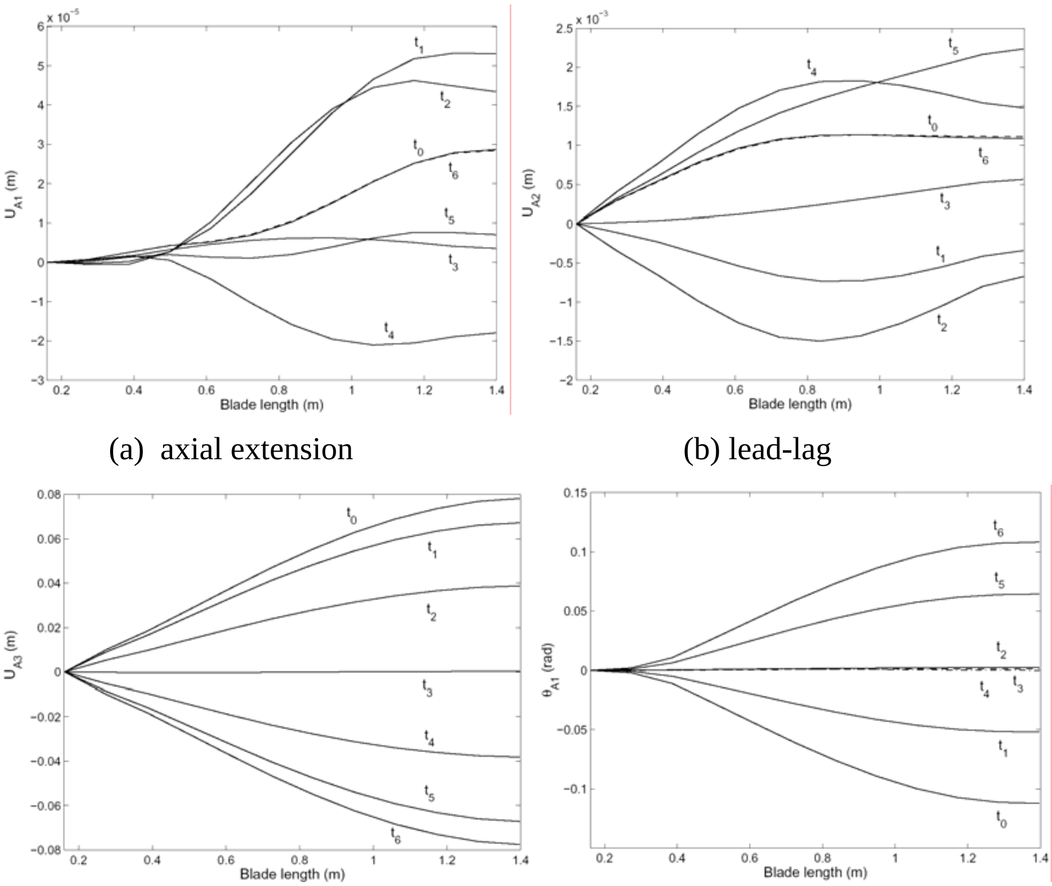
The ATR blade model used in the research is illustrated in Fig. 1. A finite element model has been developed [1] to account for the flapping, lead-lag, axial extension, and twist motions of the ATR blade model.



(a) (b)

Figure 2. The invariant manifold for the first nonlinear normal mode: the slave constraint relationships to the master coordinate.

Figure 2 shows two cross-sectional views of the invariant manifold (mode shape) for the nonlinear normal mode, where a and ϕ denote the amplitude and phase of the first flapping linear mode, respectively; S_2 and S_{11} denote the slaved amplitudes of the first lead-lag linear mode and the first axial elongation linear mode in terms of a and ϕ, respectively. There are a total of 10 similar cross-sections of the manifold, corresponding to all the linear modes that are slaved to the flapping mode.



(a) axial extension (b) lead-lag

(c) flapping motion (d) torsion

Figure 3. Blade model response on the invariant manifold of the first nonlinear normal mode, shown in terms of the system states at various times.

The nonlinear modal time responses for the various degrees of freedom of the ATR blade motion are shown in Fig. 3. Since the nonlinear mode-based reduced-order model is described by a single second-order equation, system motion can be very efficiently simulated, while retaining critical information about the dynamics of all degrees of freedom. In contrast, the reference solution, based on the truncated FEM model, requires direct time simulation of 11 complicated, coupled, second-order differential equations -- an expensive calculation. The ATR blade time responses are plotted at the following time steps: the initial time $t_0 = 0$; $t_1 = 0.0039$ s; $t_2 = 0.0078$ s; $t_3 = 0.0117$ s; $t_4 = 0.0157$ s; $t_5 = 0.0196$ s; and the ending time $t_6 = 0.0235$ s, which is approximately one-half a period of the dominant flapping motion. As can be seen, a peak-to-peak bending deflection of nearly 0.16 meters is obtained for this 1.4-meter blade (Fig. 3-b), and a peak-to-peak twist angle of about 0.2 radian (11.4 degrees) is obtained in Fig. 3-d. Although the lead-lag and the axial elongation vibrational motion amplitudes are very small in Fig. 3, the nonlinear interactions contributed from these motions are critically important to the accuracy of the combined nonlinear motion, because the blade stiffness in these directions is designed to be much larger than the stiffness in the bending and torsion directions.

2.2 Innovative numerical methodology for nonlinear modal analysis

An innovative *numerical methodology* has been developed for the construction of the nonlinear normal modes of realistic rotating rotorcraft blades. The finite element formulation is based on a two-field variational principle of the Reissner-Hellinger type, and the resulting nonlinear blade finite element model is asymmetric, and sparse in matrix form. A variant of linear modal analysis has been proposed in order to transform the blade model to a standard form that is convenient for the construction of nonlinear normal modes. The governing equations of the invariant manifold have also been modified in order to construct the nonlinear modes. The resulting numerical approach is thus highly adapted to finite element models of rotorcraft blade systems, and it enables the systematic construction of their nonlinear-mode-based reduced-order models, even for complex blades.

2.3 New computer code for the construction of nonlinear normal modes

In previous research, a computer code was developed for the construction of *single/multi-mode reduced-order models,* accounting for system dynamical behavior that involves more than a single degree of freedom (DOF). This approach had been applied successfully to an 18-DOF simplified rotor blade model with an internal resonance condition. A *new computer code* has been developed for the construction of *nonlinear-normal-mode based reduced-order models* of realistic rotating rotorcraft blade systems. The code is based on the Galerkin solution technique with the incorporation of the region partition method. It is seamlessly interfaced with the nonlinear finite element code developed for the rotorcraft blade system. Furthermore, the code is written in FORTRAN 77 and is free of commercial software. As a result, the new code can be easily implemented on different computer platforms and it is convenient for technology transfer.

2.4 Component Mode Synthesis

A nonlinear component mode synthesis technique has been developed for nonlinear structural assemblies. This approach allows one to systematically build nonlinear reduced-order models for systems that are composed of assemblies of relatively simple, but nonlinear, substructures. Both free and fixed interface methods have been developed and benchmarked on sample systems. The method is ready to be implemented for a system of coupled rotorcraft blades.

2.5 Reduced-order models for nonlinear systems under periodic external excitations

A new *computer code* has been developed for the construction of reduced-order models for nonlinear systems under *periodic external excitation*. The external forces are expressed as the solution of a set of first or second order linear differential equations. Then, the usual methodology for constructing reduced-order models in the free response case is applied to the augmented system, by choosing the variables corresponding to external forces as additional degrees of freedom. With the generated reduced-order model, large-amplitude steady-state responses of nonlinear structures can be accurately captured over a frequency range of interest. Two nonlinear example systems have been investigated here, including one two-DOF lumped-parameter model and one finite-element based beam model. The accuracy of the reduced-order models obtained has been validated by comparisons with direct time simulations of the original models.

2.6 Multi-mode based reduced-order models

Previous studies of multi-mode based reduced-order models were based on polynomial series expansions with the invariant manifold approach. These were applicable only in the weakly nonlinear regime. A new *computer code* has been developed for the construction of *multi-mode based reduced-order models* accounting for more general system dynamical behaviors, including internal resonances. With the incorporated Galerkin-based technique, the obtained reduced-order model can be accurate over chosen amplitude regions. We have validated this new code for a three-DOF example system: a four-dimensional invariant manifold has been constructed, and motions on the manifold have been reduced to four differential equations. This approach has also been applied successfully to a 18-DOF rotor blade model with an internal resonance.

2.7 Extensions of nonlinear modal analysis

Previous work on nonlinear normal modes has dealt primarily with systems with smooth nonlinearities. However, many engineering systems involve components with contact, clearance, pre-loads, friction, or different elastic materials. Such systems are often conveniently modeled by equations of motion with piecewise linear terms. Based on the concept of invariant manifolds, the model reduction technique has been successfully extended to *systems with piecewise linear terms*. The reduced-order models generated can accurately capture system motions over a wide range of amplitudes, including those with strong nonlinear effects. A two-DOF piecewise linear system has been used to illustrate the technique. The existence, stability and bifurcations of the nonlinear normal modes have also been investigated for this example.

The model reduction methodology can also be utilized for *hybrid linear-nonlinear structural systems*. Based on an example system, a linear rotating shaft supported by two nonlinear bearings, the validity of these reduced-order models has been verified. This is a necessary step for the investigation of large-scale mechanical systems connected by nonlinear joints.

2.8 Improved Nonlinear Galerkin Techniques

In previous work, the invariant manifold equations were discretized via a Galerkin expansion using basis functions in amplitude and phase coordinates and subsequent projection, yielding a set of nonlinear algebraic equations for coefficients that are solved using the Hybrid Powell method. Here, a significantly more efficient computer code has been developed for the generation of reduced-order models. In addition to the original Galerkin expansion, a *collocation projection* has been used in the new code, resulting in a 20-fold improvement in the speed of model generation. With this new code it is technically practical to construct reduced-order models for large-amplitude motions of large scale, complex structural models. This code has been tested on a number of problems, including simple academic mass-spring models and finite element beam models.

2.9 Finite element applications

These nonlinear modal analysis techniques have been developed while keeping in mind that the ultimate outcome will be user-friendly computer codes that will allow for the efficient simulation of the nonlinear vibrations of complex structures. In that vein, we have conducted a thorough investigation of the convergence of reduced-order models for a *finite-element based rotorcraft blade model*. It has been shown that the nonlinear model-based reduced-order models achieve excellent results up to large amplitudes (tip displacements about 13% of the blade length) using only a single nonlinear mode. This is crucial, since the majority of practical vibration problems are cast in finite element form. It is also unique, since most analytical techniques developed for nonlinear vibrations are not compatible with finite element formulations.

3 PUBLICATIONS

3.1 Papers published in peer-reviewed journals

D. Jiang, C. Pierre, and S. W. Shaw, "The Construction of Nonlinear Normal Modes for Systems with Internal Resonance: Application to Rotating Beams," *International Journal of Non-linear Mechanics*, Vol. 40, 2005, pp. 729-746.

D. Jiang, C. Pierre, and S. W. Shaw, "Large Amplitude Nonlinear Normal Modes of Piecewise Linear Systems," Journal of Sound and Vibration, Vol. 272, 2004, pp. 869-891.

M. Legrand, D. Jiang, C. Pierre, and S. W. Shaw, "Nonlinear Normal Modes of a Rotating Shaft Based on the Invariant Manifold Method," *International Journal of Rotating Machinery* Vol. 10, 2004, 319-335.

P. Apiwattanalunggarn, S.W. Shaw, C. Pierre, and D. Jiang, "Finite-Element-Based Nonlinear Modal Reduction of a Rotating Beam with Large-Amplitude Motion," *Journal of Vibration and Control*, Vol. 9, 2003, pp. 235-263.

E. Pesheck, C. Pierre, and S. W. Shaw, "Modal Reduction of a Nonlinear Rotating Beam Through Nonlinear Normal Modes," *Journal of Vibration and Acoustics*, Vol. 124, No. 2, April 2002, pp. 229-236.

E. Pesheck, C. Pierre, and S. W. Shaw, "A New Galerkin-Based Approach for Accurate Nonlinear Normal Modes Through Invariant Manifolds," *Journal of Sound and Vibration*, Vol. 249, No. 5, January 2002, pp. 971-993.

3.2 Papers published in conference proceedings

C. Pierre, D. Jiang, and S.W Shaw, "Nonlinear Normal Modes of a Rotating Helicopter Blade", *Proceedings of the International Conference on Nonlinear Dynamics-KPI 2004*, Kharkov Polytechnical Institute, Kharkov, Ukraine, September 14-16, 2004.

P. Apiwattanalunggarn, S.W. Shaw, and C. Pierre, "Component Mode Synthesis Using Nonlinear Normal Modes," *Proceedings of the 19th Biennial Conference on Mechanical Vibration and Noise*, Paper DETC/2003-48441, Chicago, September, 2003.

D. Jiang, C. Pierre, and S.W. Shaw, "Nonlinear Normal Modes for Vibratory Systems under Periodic Excitation," *Proceedings of the 19th Biennial Conference on Mechanical Vibration and Noise*, Paper DETC/2003-48443, Chicago, September, 2003.

D. Jiang, C. Pierre, and S. W. Shaw, "The Construction of Nonlinear Normal Modes for Systems with Internal Resonance: Application to Rotating Beams," *Proceedings of the 2002 ASME International Mechanical Engineering Congress*, New Orleans, Louisiana, November 17-22, 2002.

M. Legrand, D. Jiang, C. Pierre, and S. W. Shaw, "Nonlinear Normal Modes of a Rotating Shaft Based on the Invariant Manifold Method," *Ninth International Symposium on Transport Phenomena and Dynamics of Rotating Machinery* (ISROMAC-9), Honolulu, Hawaii, February 10-14, 2002.

D. Jiang, V. Soumier, C. Pierre, and S. W. Shaw. "Large amplitude nonlinear normal modes of piecewise linear systems," In *Proceedings of the 18th Biennial Conference on Mechanical Vibration and Noise*, ASME Paper DETC2001/VIB-21734, Pittsburgh, Pennsylvania, September 9-12, 2001

3.3 Papers presented at meetings, but not published in conference proceedings

D. Jiang, C. Pierre, and S.W. Shaw, "The Construction of Nonlinear Normal Modes for Systems with Internal Resonance: Application to Rotating Beams," Abstract., *Ninth Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures*. Blacksburg, VA, July 28-August 1, 2002.

3.4 Manuscripts submitted, but not published

P. Apiwattanalunggarn, S.W. Shaw, and C. Pierre, "Component Mode Synthesis Using Nonlinear Normal Modes," to be submitted to *Nonlinear Dynamics*, in print, 2005.

D. Jiang, C. Pierre, and S.W. Shaw, "Nonlinear Normal Modes for Vibratory Systems under Harmonic Excitation," *Journal of Sound and Vibration*, in print, 2005.

4 SCIENTIFIC PERSONNEL

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5 INVENTIONS

None.

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